Robust Bayesian inference of disparity maps
(stereo 3D and ground deformation)

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Outline

- Applications - objectives
- Problems with existing methods
- Bayesian inference
- Assumptions, forward model
- Smoothness priors
- Graphical model
- Preliminary tests
- Some results from real data
- Conclusion & future work
Summary

- **Applications: Earth & planetary sciences**
  - High-resolution ground deformation maps
  - Surface reconstruction: DEMs of natural areas

- **Our main objectives**
  - Dense vector disparity maps with **sub-pixel accuracy**
  - Provide the **uncertainties** to allow for error propagation

- **Why use optical images**
  - Availability, coverage, redundancy, price

- **Requirements**
  - **Raw images, well-sampled**

- **Necessary tools**
  - Probability theory, signal processing, computer vision, applied math, and some Physics!
Some problems with existing methods for stereo 3D reconstruction

Shape from Stereo

**Drawbacks:**
- Relies on finding point **matches** in both images
- The **density** of detected features is not uniform

Dense stereo via disparity maps

**Drawbacks:**
- Photometric **matching areas** in both images of minimum **size**
- Usually works in 1D from **resampled** images!

Generalized stereo (deformable models)

**Drawbacks:**
- Not Bayesian: difficult to estimate model parameters...

Sub-pixel accuracy, uncertainty map, 2D vector?
Infer the parameters of a dense deformation field
2 images: one before, one after earthquake/deformation/event...

- Deformation field = **spatially variable** translation vectors
- Challenge: **subpixel accuracy** (0.1 pixel to detect a 30 cm shift)
- Allow for discontinuities on segments (faults)
Tests: existing methods for ground deformation measurement

Klinger et al, 2006, Kunlun fault. 1m accuracy, 320m resolution

“optical image correlation”

Real remote sensing data

Simulations

Reference

Fourier 32x32 (correlation)

Fourier 128x128 (correlation)

Image space, nonrigid (least squares)

Klinger et al, 2006, Kunlun fault. 1m accuracy, 320m resolution

“optical image correlation”

High resolution? Uncertainty map?
Bayesian inference - basic ingredients

\[ p(\theta | \text{observations}) = \frac{p(\text{observations} | \theta) \times p(\theta)}{p(\text{observations})} \]

**OBJECTIVE:** posterior probability density function (pdf)

- parameters of interest (unknown solution)
- evidence (useful for model comparison)
- likelihood (image formation model)
- prior model (a priori knowledge about the observed object)

**Forward modeling:**
- All parameters are random variables
- Data - image formation model (rendering + degradations)
- Prior - object modeling (disparities, 3D, etc.)

**Bayesian inference:**
- Integration w.r.t. all unwanted parameters (marginalization)
- Functional optimization (deterministic for speed)
- Compute the uncertainties (covariance matrix)
- Model selection and assessment...
- Approximations required (otherwise untractable)
Common, underlying reflected radiance map

Model this map as a 2D image:

- Choose an appropriate **parametrization** and topology
  - Sampling grid size $\epsilon$
  - Rectangular lattice

- Use the **sampling theorem**
  - Frequency cut-off (optical resolution)
  - Well-sampled images (wrt. Nyquist rate)
  - Near-optimal representation using Splines:

$X = L \ast S$

**convolution**

$L = \text{set of Spline coefficients}$
The forward model

Assumptions:
- Underlying irradiance field $X = L \ast S$
- Well-sampled images
- Irradiance @ sensor prop. to reflected radiance @ surface
- Reference image $Y^1$ (zero motion, irradiance $L$)
- No occlusion (yet)

Irradiance fields before radiometric changes:
- $X^1 = X$
- $X^2 = W(X,d_x,d_y)$ warping via B-Spline interpolation

Observed (raw) images with radiometric changes and additive noise:
- $P(Y^1 \mid X^1)$
- $P(Y^2 \mid X^2)$ conditional probability distributions
Modeling radiometric changes

**Parametric**
- **Multiplicative** changes - include reflectance effects (non-Lambert, lighting variations), shadows, atmospheric attenuation, instrumental artifacts...
- **Additive** changes - include atmospheric haze, clouds, instrumental biases...
- **Additive noise** - approx. Gaussian, independent pixels

**Nonparametric**
- Includes all effects!
- More flexible
- More parameters (too many?)
Why use a spatially adaptive change model

Test - joint histograms after nonrigid registration:

No changes: diagonal

Add. and mul. changes

Test area and simulated changes

A, B or $P(Y^1,Y^2)$ should be spatially adaptive!
Simple change modeling: a single additive stochastic process

**Markov Random Field:**
spatial interactions btw. neighbor pixels

**Self-similar noise process**
- Exploits the **scale invariance of natural images** (assumption: change maps are natural images)
- Simply expressed in the **spatial domain** if integer exponent: Markov Random Fields, use image gradient operators
- **Spatially adaptive** multiplier (known in advance)
- Includes the **observation noise** (rough approximation)

**Self-similarity** of natural scenes
A smoothness prior model for disparity maps

- **Arbitrary** disparity maps: surface deformation
  - Depends on the application (earthquakes, erosion...)

- **Constrained** disparity maps: 3D reconstruction
  - Epipolar lines... *if known!*
  - $d_x$: very smooth
  - $d_y$: related to the topography
    
    ...*planetary surface modeling*

Separable prior distribution $P(d_x)P(d_y)$

Self-similar process based on image gradient operators
Bayesian inference from 2 observations

Graphical model: build the joint probability density function (pdf)

Marginalization: integrate the joint pdf w.r.t. all nuisance variables
**Integrate** prior/change parameters from the beginning

**Nonlinear, but parameter-free** method! *(PIA’07 paper)*
Method M2 - explicit hyperparameters

- Use explicit values for prior and change parameters
- Use the evidence framework to estimate them automatically, then plug in the estimated values.
How the inference algorithm works

Compute the marginal Maximum A Posteriori, and a Gaussian approximation around the optimum

Iterative optimization of a marginal energy $U$
(nonlinear conjugate gradient)

$$U = -\log P(d_x, d_y \mid Y^1, Y^2) = D(d_x, d_y, Y^1, Y^2) + \text{Prior}(d_x) + \text{Prior}(d_y)$$

- Inverse covariance matrix: **uncertainties**
  - Second derivatives of the energy $U$ at the optimum
  - Sparse matrix

compound result storage

<table>
<thead>
<tr>
<th>diagonal</th>
<th>vertical</th>
<th>diagonal</th>
</tr>
</thead>
<tbody>
<tr>
<td>optimal $dx, dy$</td>
<td>self</td>
<td>horiz.</td>
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Bayesian inference: preliminary tests
(change = iid Gaussian noise, window-based estimation)

Bayesian inference: disparity dx,dy
variance of dx,dy
Bayesian inference: disparity dy + error bars
(and ground truth)

Bayesian inference:
disparity dy + error bars
(and ground truth)

reference dy
source image
Results - real data, inference method M1

Spline-subsampled (8x) HRSC stereo s1s2, 64x64 pixels, 1 disparity vector / 4x4 pixels

Bayesian inference:
estimated disparity dx [0,9], dy [3.5,-1.5]

standard deviation maps [0,0.3]

change map [-5,5]

images Y1, Y2 [0,255]

(PIA’07 paper)
Results - real data, inference method M2

RAW SPOT 5, multidate, 128x128 pixels @ 3.5m, 1 disparity vector / pixel

Bayesian inference:
estimated disparity \(dx,dy\) [-1,1]
change map
[-10,10]
standard deviation maps [-0.2,0.2]
correlation map [-1,1]
images \(Y1, Y2\) [0,255]
color map
Results - simulations, inference method M2

Bayesian inference:
estimated disparity $dx, dy \in [-1, 1]$

true disparity map $dx, dy \in [-1, 1]$  
true changes $[-10, 10]$  
images $Y1, Y2 [0, 255]$  

color map

Bayesian inference:  
estimated disparity $dx, dy \in [-1, 1]$  

change map $[-10, 10]$  

stddev maps $[-0.1, 0.1]$  

correlation map $[-1, 1]$  

uncertainties

64x64 pixels, 1 disparity vector / pixel
Conclusions

Accomplishments
- High-resolution (pixel level), accurate disparity maps
- Robustness to radiometric changes
- Parameter-free and automated methods
- Uncertainty estimation (error maps): added value
- Fast, multi-grid implementation

In progress
- Data term: spatially adaptive, robust to occlusions
- Prior term: better disparity models (curvature)
Future work

To do...
- Push-broom **camera calibration** from the disparity map
- Convert the disparity map into an **elevation model**

- Full 3D surface recovery **from n images**:
  - Rendering: take into account possible **occlusions**
  - **Reflectance** map inference

- **Validation** on real data (raw images required)
  - Along-track (simultaneous): HRSC on Mars Express, ASTER
  - Across-track (multidate): SPOT 5
  - Ground truth? LIDAR points, RADAR DEM...
Effects of resampling via interpolation
...or why use raw images!

**Shift only** (true shift = 0,0) → **systematic errors**

Accuracy vs. type of interpolation:

Uncorrelated pixels (raw image) become **correlated** after geometric resampling (e.g. orthorectification)

→ the rigorous evaluation of uncertainties requires raw data!