Bayesian Vision for Surface Recovery

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Bayesian Vision

- **Computer vision**
  (model reconstruction from multiple observations):
  *inverse problem of rendering*

- **Bayesian inference**
  applied to this inverse problem:
  everything is described by *random variables*

- Data fusion into a single model becomes a
  *parameter estimation* problem

- It can be solved by existing efficient
  *optimization* techniques
• Problem statement, main contributions
• Generative model & posterior pdf
• The image formation model
• Surface and sampling models
• Bayesian inference, marginalization
• The proposed algorithm
• Results and conclusions
• Extension to 3D shape recovery
• Future work
• Appendix: Derivative computation
Problem statement

- **n>2 noisy images** of a textured surface recorded by **n calibrated** cameras

2D experimental setting

Recover the surface **geometry** and the **uncertainty** given the observed images

**Assumptions:**

- **Similar lighting conditions** (stereo setting)

- **Change of variables:**
  - **irradiance** \( L = \text{albedo} \times \text{reflectance} \\
  \text{does not depend on the viewing conditions}
Contributions summary

- **Model of the unknown object (surface)**
  - Choice of an appropriate **parametrization** and topology
  - Use efficient **priors** to help stabilize the problem

- **Observed image formation model**
  - Understand the **image formation** and degradation processes
  - Choice of an appropriate **discretization** scheme

- **Efficient Bayesian inference scheme**
  - **Separate** and **marginalize out** the irradiance (texture)
  - Find **approximations** to make it computationally tractable
The (forward) generative model
multiple observations

- **Surface** v, L
  - Mean: initial estimate (if any)
  - Uncertainty on this estimate
  - Smoothness prior

- **Observations** \{X^i\}
  - Mean: rendering I(v, L, \Theta)
  - Uncertainty from noise N(0, \sigma^2)

- **Observation parameters** \{\Theta^i\}
  - Mean: calibration
  - Uncertainty on camera pose

**Graphical model:**
relations between random variables
(conditional densities and hyperpriors)
Building the joint posterior pdf

\[
P(\text{surface, cameras} \mid \text{images}) \propto P(\text{surface}) \times \prod_i P(\text{camera}_i) \times \prod_i P(\text{image}_i \mid \text{surface, camera}_i)
\]

- **Posterior density**
  \[\{v^k\}, \{L^j\}, \{\Theta^i\}, \{X^i\}\]

- **Prior density**

- **Likelihood**

**Surface model**
- Gaussian pdf
  (smoothness prior)
  - Geometry \(v\)
  - Irradiance \(L = \rho f\)

**Camera prior**
- Dirac pdf
  (calibrated camera)
  - Camera pose \(\Theta\)
  - Camera physics

**Image formation model**
- Gaussian pdf
  - Rendering \(I(v, L, \Theta)\)
  + additive Gauss. noise
  \(N(I(v, L, \Theta), \sigma^2)\)
The image formation graphical model for a single image

- **Likelihood:** \( P(X \mid v, L, \Theta) = Z_0^{-1} e^{-U(v,L)} \)
  
  where \( U(v,L) = \sum_p (I_p(v,L,\Theta) - X_p)^2/2\sigma^2 \)

- **Deterministic image formation:** \( I \) as a function of \( v, L \) and \( \Theta \)
  "rendering"
2 Possible surface models
parametrization and topology

Geometry

Topology: subdivided segments, coarse vertices $v^k$, fine vertices $v^j$

Parametrization: height field $v^k = (x^k, z^k)$

• Piecewise constant irradiance

Irradiance: constant $L^j$ between fine vertices $v^j, v^{j+1}$

• Piecewise linear irradiance

Irradiance: linear $(1-t) L^j + t L^{j+1}$ between fine vertices $v^j, v^{j+1}$
The geometry and irradiance pdfs

**Geometry**
Smoothness prior (surface regularity)

\[ P(v) = Z_{\alpha}^{-1} e^{-\alpha \Phi(z)} \quad \text{where} \quad \Phi(z) = \sum_k (z^{k+1} - z^k)^2 \]

*Helps stabilize the problem when insufficient data (hidden surfaces or super-resolution)*

**Irradiance**
Smoothness prior (texture regularity)

\[ P(L) = Z_{\beta}^{-1} e^{-\beta \Phi(L)} \quad \text{where} \quad \Phi(L) = \sum_j (L^{j+1} - L^j)^2 \]

*Helps reduce the uncertainty on the irradiance (hidden surfaces or super-resolution): useful for marginalization*
The deterministic image formation and 2 possible sampling models

Projection 2D→1D

Pixel integration scheme
Convolution by a Point Spread Function (PSF) $h$ followed by sampling on a discrete pixel grid

$$I_p = (L*h)_p = \sum_j \lambda^j_p L^j$$

where $\lambda^j_p = \text{fct. of } P(v, \Theta)$

- **Piecewise constant PSF (discontinuous)**
  discrete combinations of “box” sampling kernels

- **Piecewise linear PSF (continuous)**
  discrete combinations of “hat” sampling kernels
Choice of the surface and imaging models

- log-likelihood: \( U(v,L) = \sum_p (I_p(v,L,\Theta) - X_p)^2 / 2\sigma^2 + \text{const.} \)

Variation of a vertex height \( z^k \) \( \Rightarrow \) variation of \( U \), derivative continuity?

### Irradiance model

- \( p. \text{ constant } L \)
- \( p. \text{ constant } \text{PSF} \)

#### Non-boundary vertex

- \( p. \text{ constant} \Rightarrow \text{discontinuous derivatives} \)

- \( \Rightarrow p. \text{ linear irradiance} \)

### Sampling model

- \( p. \text{ constant } \text{PSF} \)
- \( p. \text{ linear PSF} \)
- \( p. \text{ constant } L \)

#### Boundary vertex

- \( p. \text{ constant} \Rightarrow \text{discontinuous derivatives} \)

- \( \Rightarrow p. \text{ linear sampling kernel} \)
Bayesian inference
inverting the generative model

Compute the posterior marginal:
\[ P(v \mid \{X^i\}) = \int L \int_{\{\Theta^i\}} P(v, L, \{\Theta^i\} \mid \{X^i\}) d\{\Theta^i\} dL \]

Then find a Gaussian approximation
→ geometry estimate (mean) and uncertainty

- **Camera marginalization:** Dirac dist., known \{\Theta^i\}
- **Irradiance marginalization:**
\[ P(v \mid \{X^i\}) = \int_{\Omega} P(v, L \mid \{X^i\}) dL \propto P(v) \int_{\Omega} e^{-U(v, L) - \beta \Phi(L)} dL \]

\[ U'(v) = \text{marginal energy} \]

**Around the MAP:**
Gaussian approx. of marginal
(quadratic approx. of \(U'(v)\))

\[ \hat{v} = \arg\min_v [U'(v)] \rightarrow \text{optimization} \]

\[ \Sigma^{-1}_{kl} = \frac{\partial^2}{\partial Z^k \partial Z^l} [U'(v)]_v \]
Marginalization and approximations

\[
\int_{L} e^{-U(v,L) - \beta \Phi(L)} \, dL
\]
\[f(L|v) \propto P(L|v,\{X^i\})\] posterior pdf. of \(L\) given \(v\)

→ **Gaussian pdf:** \(f(L|v)\) (usually Laplace approximation)

\[
\hat{L}(v) = \arg \max L P(L|v,\{X^i\}) = \arg \min_{L} \left[ U(v,L) + \beta \Phi(L) \right]
\]

\[
\Xi^{-1} = \frac{\partial^2 L}{\partial L^i \partial L^j} \left[ U(v,L) + \beta \Phi(L) \right]_{L=\hat{L}(v)}
\]

\[
P(v | \{X^i\}) \propto P(v) \prod_{i=1}^{n} \left[ f(\hat{L}(v)|v) \right]^{1/2}
\]

Closed-form approx.
\[
\hat{L}(v) \approx \sum_{i,p} \frac{(\lambda_{p}^i)^{i} X_p^i}{\sum_{i,p} (\lambda_{p}^i)^{i}}
\]

**Constant covariance approximation**

Marginal energy:
\[
U'(v) = U(v,\hat{L}(v)) + \beta \Phi(\hat{L}(v)) - \log P(v)
\]
The optimization algorithm

Compute the marginal MAP and a Gaussian approx. around it

Iterative optimization of the marginal energy $U'(v)$

- **Irradiance marginalization**: compute $\hat{L}(v)$
- **Linearize** the rendering around $\bar{v}$ using the derivatives:

$$I_p(v,\hat{L},\Theta) \approx I_p(\bar{v},\hat{L},\Theta) + \sum_k \left[ \frac{\partial I_p}{\partial v^k} \right]_{\bar{v}} (v^k - \bar{v}^k)$$

$$\Rightarrow U'(v) = U(v,\hat{L}) + \beta \Phi(\hat{L}) + \alpha \Phi(v) \quad \text{approx. by a quadratic form}$$

**1st order Markov** structure for $v \Rightarrow \textbf{ICM}$ optimization method (*no CG*)

In practice, a **parallel quasi-Newton** method can be used

- **Update**: one step is sufficient  \( \text{(quadratic approx. not very accurate!)} \)

$$\tilde{v}^k \leftarrow \tilde{v}^k - \left[ \frac{\partial U'}{\partial v^k} \right]_{\tilde{v}} \left[ \frac{\partial^2 U'}{\partial v^k \partial v^k} \right]_{\tilde{v}}^{-1}$$

- **Repeat** until convergence, then compute uncertainty $\left[ \Sigma^{-1} \right]_{kl} = \left[ \frac{\partial^2 U'}{\partial v^k \partial v^l} \right]_{\bar{v}}$
Results of the 2D experiment

Diagonal quasi-Newton, 10 iterations

inferred geometry \( P > 0.1 \)
• Main ideas:
  - Bayesian approach: invert the image formation in 2D
  - Change of variables (irradiance = albedo x reflectance)
  - Use continuous models for both irradiance and PSF
  - Marginalize out the irradiance using approximations
  - Use a quasi-Newton method to infer the solution

• Conclusions of the experiments:
  Marginalization:
  - Reduces problem dimension / dependence structure
    (independent vertex optimization, uncertainty computation)
  - Makes the energy landscape more quadratic
    (efficient quasi-Newton ICM approach)

Irradiance and PSF continuity:
Essential for energy landscape smoothness / derivative continuity
(optimization stability and efficiency)
• **Extension to 3D (in progress):**

  – **Visibility computation**
    hidden surfaces: recursive polygonal approach
  – **Continuous irradiance**
    moments (order 2) of the visibility polygons
  – **Continuous PSF**
    quadrant functions (separable hat function)
  – **Complex BRDFs** ($\neq$ Lambert)
    robust irradiance marginalization (unknown camera-dependent factor)

• **Future extensions:**

  – Multispectral albedo
  – Different lighting conditions
  – Adaptive subdivision
  – Adaptive BRDFs
  – Other types of camera
Future research - 3D modeling and inference

• **3D surface modeling:**
  
  Study real surfaces (Earth, Mars)
  – Reflectance/geometry **interactions**
  – **Multispectral** reflectance

• **Bayesian surface reconstruction:**

  **Marginalization:**
  – First infer the **geometry** (irradiance marginalization)
  – **Albedo** inference using estimated geometry
  – Simultaneous **reconstruction/calibration**

  **Bayesian model selection:**
  – Infer the **topology** (overhangs, etc.)
  – Dynamic and **adaptive subdivision**